

# Artillery Scheme Optimization Based on Combination Weighting Method

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**Abstract.** System efficiency method is widely used in the evaluation and selection of multiple schemes for equipment designing in the stages of preliminary demonstration and schematic design. In order to satisfy the demand of multi-scheme optimization for new equipment by selecting artillery with different calibers, in this paper, we analyze the operating requirements and the combat environment of artillery, and develop an inherent capability evaluation system composed of firing capability and damage ability. Due to the different preferences of experts for rating index weights, we have further considered the combination of subjective and objective weighting method in the artillery multi-scheme evaluation model. The results of numerical experiments show that proposed combination weighting method can not only avoid the different preferences of experts, but also reduce the deviation from the actual demands.

**Keywords:** system efficiency method; analytic hierarchy process; entropy weight method; scheme optimization; weight

## 1. Introduction

Artillery, as the backbone of conventional warfare, has the distinctive features of strong continuous combat capability, rapid reaction speed, low life cycle cost and wide variety of ammunition [1], and therefore is widely used in all the arms and services. As early as the beginning of World War I, machine guns were first installed on combat aircraft; by World War II, artillery was gradually installed on combat aircraft, with a caliber range of 20 mm, 23 mm, 25 mm, 27 mm, 30 mm, 37 mm and so on, and has been highly valued by all countries and widely equipped in various types of combat aircraft [2]. Artillery equipped in combat aircraft are mainly used in air combat (especially close combat) and ground attack, for example, artillery equipped in attack aircraft can offer direct support for ground combat troops.

A new type of equipment demanding continuous close-range ground fire support, is proposed to select small-caliber artillery as one of the main weapons. Current small-caliber aircraft guns have only a short firing range and single ammunition type, and are mainly used in air combat, but not applicable to ground attack. Considering requirements for the equipment development cost and cycle, several mature small-caliber ground artillery with adaptive improvement are chosen as alternative artillery. Due to the difference in the performance of the alternative artillery, the comparison and optimization of the scheme become the focus. With the system efficiency analysis method, multi-scheme evaluation and optimization can be carried out to find the best solution to meet the requirements. In this paper, the weight is determined with the method of the combination of subjective and objective weighting, and the inherent capability of each alternative is analyzed and compared, in order to provide support for the artillery scheme optimization.

## 2. Inherent Capability Evaluation System

Taking the commonly used ADC model as an example, and assuming that the system is in good condition and faultless, the inherent capability can be considered as the only factor in the multi-scheme comparison phase. Inherent capability is the performance of a known system in the mission, a measure of the capability of a weapon system to accomplish a specified task, and a comprehensive measure of the performance, target characteristics, and combat environment of a weapon system [3].

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There has been a lot of research on the artillery efficiency index system. Reference [4] has established an index system of firepower strike capability consisting of autonomous combat capability, firing capability, damage ability, etc., and [5-7] have established an index system of firepower strike capability, mainly consisting of reaction capability and damage ability. Reference [8] and others have established a fire assault index system consisting of power, lethal area, firing range and firing rate. Reference [9] has established a ready-to-fire capability index system consisting of the rapid reaction capability, ammunition supply capability and firepower coverage capability. However, due to great changes in the operating requirement and combat environment in this paper, the artillery efficiency index systems mentioned above cannot be directly applied, therefore a multi-scheme evaluation system of the artillery in the new equipment should be established by referring to the above systems.

## **2.1. Firing Capability**

Considering the index difference and operating requirements of the alternative artillery for the new equipment, the following index are proposed to evaluate the firing capability.

1) Maximum effective firing range. The firing range is determined by the muzzle velocity of the projectile, the ballistic coefficient (determined by projectile mass, shape coefficient, etc.) and firing angle, etc., and the firepower characteristics can be represented by this index. Firstly the alternative artillery launches the main projectile horizontally at a certain flight altitude, then taking the situation when the projectile's ballistic storage speed in the standard atmosphere falls to Mach 1.1 as a ballistic termination condition, the firing altitude and maximum effective firing range of the alternative artillery is calculated through repeated iterations. The maximum effective firing range obtained from the above ballistic calculation is taken as the characteristic value of the index.

2) Maximum firing rate. Alternative artillery's automatic mechanisms work differently and thus have different firing rates. Alternative artillery index value can be selected as the characteristic value of the evaluation index.

3) Ammunition carrying capacity. The total weight requirement for artillery and ammunition is determined in the new equipment. Since the weight of alternative artillery and its adaptive ammunitions varies, the maximum allowable ammunition carrying capacity (only for the main type) when the total weight is certain, can be selected as an evaluation index.

4) Number of ammunition types. This equipment is required to possess a strong close-range fire support capability so it can strike vehicles, effective strength, etc. , consequently it requires alternative artillery to launch a variety of ammunition. The number of ammunition types can be selected as an index of the ability to strike multiple types of targets.

## **2.2. Damage Ability**

According to the operating requirement and the task capability mapping table of the new equipment, ground vehicles, effective strength and so on are typical targets to strike. By calculating the damage probability or lethal proportion of alternative artillery to typical ground targets at different firing altitudes, the characteristic value of damage ability can be obtained.

1) Damage ability of striking ground vehicle targets. Hummer is selected as the typical target. Its damage condition and criteria are determined according to the mechanism of fragment damage, and damage probability at different firing altitudes is calculated.

2) Damage ability of striking ground effective strength targets. The exposed standing posture personnel is selected as the typical target, distribution characteristic of which is detailed in [10]. Its damage condition and criteria are determined according to the mechanism of fragment damage, and lethal proportion at different firing altitudes is calculated.

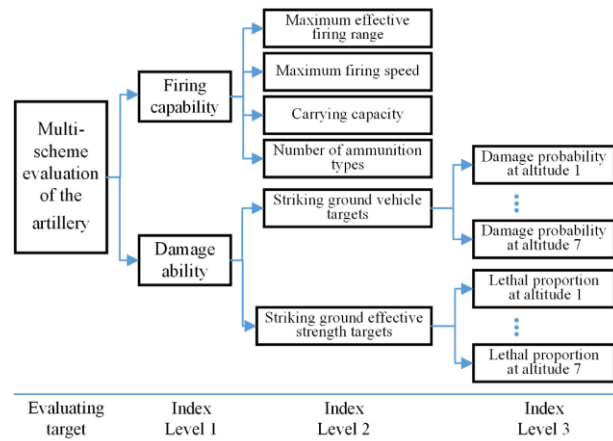


Fig. 1. Inherent capability evaluation system of artillery scheme

### 2.3. Evaluation System

By combining the above analysis, an inherent capability evaluation system of the artillery scheme applicable to the new equipment can be established, as shown in Fig. 1.

## 3. Establishment of Artillery Scheme Optimization and Evaluation Model

The index weight assignment of the evaluation system is one of the key links in the evaluation of system efficiency, thus whether the weighting method is reasonable will directly affect the evaluation results [11]. At present, according to the source of the original data and the weight calculation method, the weighting method can be approximately divided into subjective weighting method and objective weighting method. Subjective weighting method is mainly calculated by expert's preference information or empirical judgment of the index, such as analytic hierarchy process, Delphi method, etc. The objective weighting method is mainly calculated by index data information, such as entropy weighting method, principal component method and margin maximization method [3,11,12,13].

While the cognition of this new equipment is still deepening, the evaluation index has both artillery performance index, but also involves the installation constraints of aviation platform. As experts on aviation, weapons and other related industry evaluate the weight of the index with great difference, and the single use of subjective weighting method makes the weight difficult to determine and lack of objectivity. It is proposed to use the combination of subjective weighting method and objective weighting method to determine the weight of each index, and strive to fully consider subjectivity while ensuring its objectivity.

### 3.1. Weight Determination by Analytic Hierarchy Process

Analytic hierarchy process is a typical subjective weighting method, which is widely used in system efficiency analysis. Its main steps are as follow.

#### 3.1.1. Building an index judgment matrix

In consideration of the cumulative effect and the degree of loss of judging information, etc. caused by the scale method, the experts are usually investigated by the "9 scale method".

Methods of proportion scale [3] and exponential scale [11] are often used to establish judgment matrix  $A$ . The commonly used proportion scale is used in this paper, to discuss and determine the judgment matrix by a number of experts from related industries such as aviation and weapons.

#### 3.1.2. Weight calculation

The largest eigenvalue  $\lambda_{max}$  and eigenvector  $W_A$  of judgment matrix  $A$  are obtained by square root method [14], and the calculation process will not be repeated here.

#### 3.1.3. Consistency test

Calculating consistency index  $CI$  as,

$$CI = \frac{\lambda_{max} - n}{n - 1} \quad (1)$$

where,  $n$  is the order to judgment matrix.

Then the consistency ratio  $CR$  is,

$$CR = \frac{CI}{RI} \quad (2)$$

where,  $RI$  is an average random consistency index, as detailed in [3].

For the judgment matrix greater than order 2, the consistency test can be accepted generally when  $CR \leq 0.1$ .

Due to the ambiguity and uncertainty of the judgment, as well as the subjectivity of experts, the judgment matrix may not pass the consistency test. In this case, the judgment matrix needs to be further adjusted until the consistency test requirements are met. This process will not be covered here.

### 3.2. Weight Determination by Entropy Weighting Method

Entropy is a degree to measure system's disorder in information theory, that is, the more orderly a system is, the smaller its entropy value is. For an index, the greater the variation of the sample value is, the smaller the information entropy will be, indicating that more information is provided and the greater the role is played in the comprehensive evaluation; on the contrary, the smaller the variation of the sample value is, the greater the information entropy will be, indicating that less information is provided and the smaller the role is played in the comprehensive evaluation [15].

When using the entropy weight method, the information of the sample size of the index is used to judge the validity and value of the index, the main steps are as follows:

#### 3.2.1. Data standardization

For the evaluation matrix  $X^*$  with sample number of  $m$  and index numbers of  $n$ , matrix  $X$  is obtained by standardized treatment according to formula (3),

$$\begin{aligned} x(i, j) &= \frac{x^*(i, j) - x_{min}(i, j)}{x_{max}(i, j) - x_{min}(i, j)} \quad \text{forward indicator} \\ x(i, j) &= \frac{x_{max}(i, j) - x^*(i, j)}{x_{max}(i, j) - x_{min}(i, j)} \quad \text{reverse indicator} \end{aligned} \quad (3)$$

where,  $x^*(i, j)$  is the  $j$ th index value of the  $i$ th sample,  $x_{max}(i, j)$  and  $x_{min}(i, j)$  are the maximum and minimum values of the  $j$ th index value respectively.

#### 3.2.2. Calculate the entropy value of the index

According to the definition of entropy, the entropy value of an index is  $h_j$ ,

$$f_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}} \quad (4)$$

$$h_j = -\frac{1}{\ln m} \sum_{i=1}^m f_{ij} \ln f_{ij} \quad (5)$$

wherein, when  $f_{ij} = 0$ , then specified  $f_{ij} \ln f_{ij}$ .

#### 3.2.3. Determine the weight of the index

The entropy values of each index are calculated separately, and the weight  $W_{Bj}$  is determined as,

$$W_{Bj} = \frac{1 - h_j}{n - \sum_{j=1}^n h_j} \quad (6)$$

### 3.3. Combined Weights

There are two combinations of subjective weights and objective weights.

#### 3.3.1. Weighted average method

Reference [16] established an optimized combination weighting model, constructed the Lagrange function, and obtained the formula for calculating the combined weights,

$$W_1 = \alpha W_A + (1 - \alpha) W_B \quad (7)$$

where,  $\alpha$  is experience decision factors.

When the decision maker is inclined to objective data, the range of  $\alpha$  value is [0, 0.5], and when the decision maker is inclined to subjective experience, the range of  $\alpha$  value is (0.5, 1].

### 3.3.2. Product method

According to [12,14,17] and so on, the combined weights are calculated by product,

$$W_2 = \frac{W_A W_B}{\sum_{j=1}^n W_A W_B} \quad (8)$$

### 3.4. Calculate the Evaluation Results

The dimensional index value is normalized, mainly referring to the four index of shooting capability.

Taking into account the particularity of the evaluation model in this paper, the experts discuss and determine that if the number of evaluation index in some level is too small ( $n = 2$ ), the weight assignment is carried out directly, and if the number of evaluation index in some level is more than 2, the inherent capability is calculated through linear weighting model, at last the decision-making values of each scheme are obtained level by level.

TABLE I. EVALUATION DATA OF ARTILLERY MULTI-SCHEME OPTIMIZATION

Index Level 1	Index Level 2	Index Level 3	Scheme A	Scheme B	Scheme C	Scheme D	Scheme E
Firing capability	Maximum effective firing range		0.0000	0.1852	0.3704	0.7407	1.0000
	Maximum firing rate		1.0000	0.1688	0.0597	0.2208	0.0000
	Ammunition carrying capacity		1.0000	0.4768	0.3882	0.1066	0.0000
	Number of ammunition types		0.0000	0.6667	0.0000	1.0000	0.0000
Damage ability	Striking ground vehicle targets	Altitude 1	0.9452	0.9474	0.9552	0.9618	0.9663
		Altitude 2	0.5450	0.5609	0.5638	0.5579	0.5611
		Altitude 3	0.0000	0.0000	0.3098	0.2950	0.2890
		Altitude 4	0.0000	0.0000	0.2078	0.2000	0.1985
		Altitude 5	0.0000	0.0000	0.0000	0.1219	0.1193
		Altitude 6	0.0000	0.0000	0.0000	0.0936	0.0891
		Altitude 7	0.0000	0.0000	0.0000	0.0854	0.0760
	Striking ground effective strength targets	Altitude 1	0.4824	0.6217	0.7730	0.8121	0.9050
		Altitude 2	0.4012	0.5729	0.7186	0.7766	0.8639
		Altitude 3	0.3405	0.4531	0.5942	0.6827	0.7734
		Altitude 4	0.0000	0.3411	0.4664	0.5492	0.6565
		Altitude 5	0.0000	0.2590	0.3634	0.4500	0.5461
		Altitude 6	0.0000	0.0000	0.2952	0.3607	0.4513
		Altitude 7	0.0000	0.0000	0.2549	0.3093	0.3971

## 4. Artillery Multi-Scheme Optimization and Comprehensive Evaluation of Inherent Capability

### 4.1. Data Collection and Standardization

Data of each alternative artillery is obtained according to the evaluation index (in Fig. 1), and then is normalized and standardized. The result is shown in Table I.

### 4.2. Weight Determination

According to the method above, subjective weights, objective weights and combination weights are respectively calculated for index of each level. For there are only 2 index in Level 1, according to experts'

discussion results, the weights are determined to be 0.4 and 0.6. And also there are only 2 index in Level 2 of damage ability, according to combat operation mode of the new equipment, the weights are determined to be 0.4 and 0.6 through experts' discussion.

The firing capability judgment matrix is,

$$\begin{bmatrix} 1 & 3 & 2 & 4 \\ & 1 & 1/3 & 1/2 \\ & & 1 & 3 \\ & & & 1 \end{bmatrix}$$

The judgment matrix of damage ability of striking ground vehicle targets at different altitude is,

$$\begin{bmatrix} 1 & 1/3 & 1/3 & 1/4 & 1/7 & 1/7 & 1/5 \\ & 1 & 1/3 & 1/4 & 1/5 & 1/5 & 1/3 \\ & & 1 & 1/3 & 1/5 & 1/5 & 1/3 \\ & & & 1 & 1 & 1 & 3 \\ & & & & 1 & 1 & 4 \\ & & & & & 1 & 3 \\ & & & & & & 1 \end{bmatrix}$$

The judgment matrix of damage ability of striking ground effective strength targets at different altitude is,

$$\begin{bmatrix} 1 & 1/3 & 1/5 & 1/7 & 1/7 & 1/3 & 1 \\ & 1 & 1/3 & 1/4 & 1/4 & 1/3 & 2 \\ & & 1 & 1 & 1/2 & 1 & 3 \\ & & & 1 & 1 & 1 & 5 \\ & & & & 1 & 1 & 3 \\ & & & & & 1 & 1 \\ & & & & & & 1 \end{bmatrix}$$

When the combination of subjective weights and objective weights are calculated by (7), the number of experience decision factor is  $\alpha = 0.5$ .

The weights calculation result is shown in Table II. Weights comparisons are shown in Fig. 2 to Fig. 4.

TABLE II. WEIGHTS COEFFICIENT OF EVALUATION FOR MULTI-SCHEME OPTIMIZATION OF ARTILLERY

Index Level 1	Index Level 2	Index Level 3	Subjective weights	Objective weights	Combined weights 1	Combined weights 2
Firing capability 0.4	Maximum effective firing range		0.4617	0.1572	0.3508	0.3094
	Maximum firing rate		0.1013	0.2795	0.1368	0.1904
	Ammunition carrying capacity		0.3038	0.1812	0.2662	0.2425
	Number of ammunition types		0.1333	0.3821	0.2462	0.2577
Damage ability 0.6	Striking ground vehicle targets 0.4	Altitude 1	0.0290	0.0948	0.0161	0.0619
		Altitude 2	0.0470	0.0524	0.0144	0.0497
		Altitude 3	0.0671	0.1155	0.0454	0.0913
		Altitude 4	0.2162	0.1155	0.1463	0.1658
		Altitude 5	0.2709	0.2071	0.3288	0.2390
		Altitude 6	0.2600	0.2072	0.3156	0.2336
		Altitude 7	0.1097	0.2075	0.1334	0.1586
	Striking ground effective strength targets 0.6	Altitude 1	0.0381	0.1180	0.0335	0.0780
		Altitude 2	0.0726	0.1137	0.0617	0.0932
		Altitude 3	0.1784	0.1299	0.1731	0.1542
		Altitude 4	0.2317	0.1022	0.1768	0.1669
		Altitude 5	0.2625	0.1056	0.2070	0.1841
		Altitude 6	0.1565	0.2149	0.2512	0.1857
		Altitude 7	0.0601	0.2156	0.0968	0.1379

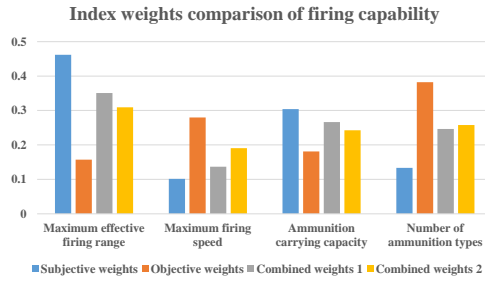


Fig. 1. Index weights comparison of firing capability in Level 2

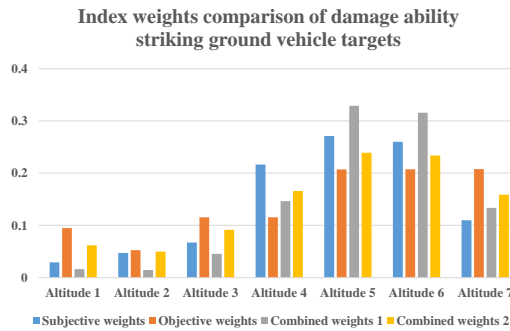


Fig. 2. Index weights comparison of damage ability striking ground vehicle targets in Level 3

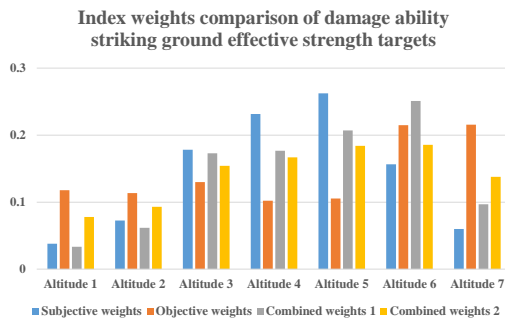


Fig. 3. Index weights comparison of damage ability striking ground effective strength targets in Level 3

### 4.3. Evaluation Results Calculation

From the above data, the evaluation results of different weights can be obtained, as shown in Table III.

TABLE III. EVALUATION RESULTS OF ARTILLERY WITH MULTI-SCHEME

	Subjective weights	Objective weights	Combined weights 1	Combined weights 2
Scheme A	0.2137	0.2655	0.2027	0.2396
Scheme B	0.2530	0.2891	0.2467	0.2710
Scheme C	0.3097	0.2649	0.2675	0.2873
Scheme D	0.4468	0.4769	0.4426	0.4618
Scheme E	0.4531	0.3402	0.3901	0.3966

As it can be seen from Table III, when the subjective weighting method is singly used, the evaluation results are  $E > D > C > B > A$ , which are fully in line with the subjective judgment that the larger artillery caliber is, the greater its firing capability and damage ability are. However, Scheme E is not superior in terms of firing rate, ammunition carrying capacity and number of ammunition types, so experts' preference for maximum effective range will lead to decision bias. At the same time, even if the weights of the index in Level 1 are adjusted, there still exists the problem that evaluation results of scheme E and D are close, therefore it is inappropriate to select a scheme with relative advance.

When the objective weighting method is singly used, the evaluation results are  $D > E > B > A > C$ . Scheme D is superior in several index of firing capability, so its evaluation results can win by relative

advantage. However, because the data in the maximum rate index of scheme A is much higher than the other schemes, the data difference between the samples is large, causing the weight of the maximum rate index much higher. Through analyzing the unique combat mode of the new equipment, and considering that ground strike targets are mostly in static or low-speed motion state, the influence of the firing rate on the combat effect is not that significant. Therefore, considering the actual combat needs, the weight of the maximum firing rate index should not be high.

Through combining subjective weights and objective weights by (7) and (8), the evaluation results are  $D > E > C > B > A$ , which have, not only avoided the preferences of experts in different industries, but also avoided that the weight does not meet the actual combat demands due to large data difference between the samples.

## 5. Conclusion

In the preliminary demonstration and schematic design stage when multi-scheme optimization is proceeding, the system efficiency method can be used to evaluate and to select the schemes, in order to find the best scheme to meet the requirements. In this paper, taking a new type of equipment equipped with different small-caliber artillery as an example, referring to the artillery efficiency index system, a multi-scheme inherent capability evaluation index system is constructed, and an evaluation model of combining subjective weights and objective weights is established, to evaluate and sort the inherent capability of each scheme.

Through practical example analysis, when subjective weights are singly used, it is inappropriate to select a scheme with relative advance, due to experts' preferences for the maximum effective range; when objective weights are singly used, the data difference in the maximum firing rate index between the samples is large, which is not in line with the actual combat needs; when combined weights are used, the evaluation results not only avoid experts' preferences, but also avoid the situation that the weights do not meet the actual demands due to large data difference between the samples. It is shown that the research approaches can be used for those similar multi-scheme optimization problems. In fields of military, architecture, environment, etc., combination weighting method is suitable for their evaluation and selection of the multiple schemes.

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